What Does Low Math Performance Mean?

This paper suggests that an overwhelming majority of my Geometry students cannot perform on Geometry assessments because they do not even understand the questions being asked.

Purpose of this Paper

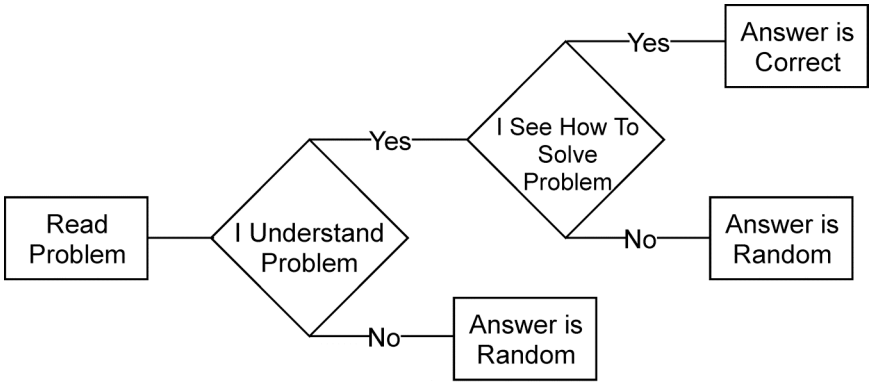
This paper reports preliminary results of an analysis I am performing as part of my annual goal for the school year 2006-2007.

The question this paper addresses can be stated this way:

> To what extent is what we observe as low performance in math tests due to students' inability to understand problem statements?

The Problem-solving Model

This model of the process a student uses to answer a multiple-choice question is implicit in the analysis.



The second decision box, "I see how to solve problem" is where we as math teachers put our energy. We understand that there is a "reading comprehension" issue that affects the answer to the "I understand problem" decision box, but mitigating that issue is not where our skills or training are directed. Beyond our general obligation to provide reading and writing across the curriculum, we do not address this issue.

My belief, based on observation of how my Geometry students attack their Geometry problems, has been that a significantly high fraction of my students do not even reach the "I see how to solve problem" decision box because they do not understand the problem statement. The purpose of the work on which this paper reports has been to test this belief.

Gathering Data

I designed and administered a ten-question multiple-choice test that asks students direct questions about their understanding of Geometry problem statements. The test appears as the Appendix to this report. The correct answer to each question and the fraction of students that selected each of the four choices have been added to the left margins of the test.

The problem statements about which the students' understanding was being examined were all taken (with some liberties in problems 5 and 6) from worksheets and tests that are part of the Glencoe Geometry textbook system. The students had been asked to solve all these problems as part of their normal Geometry curriculum before they took this test.

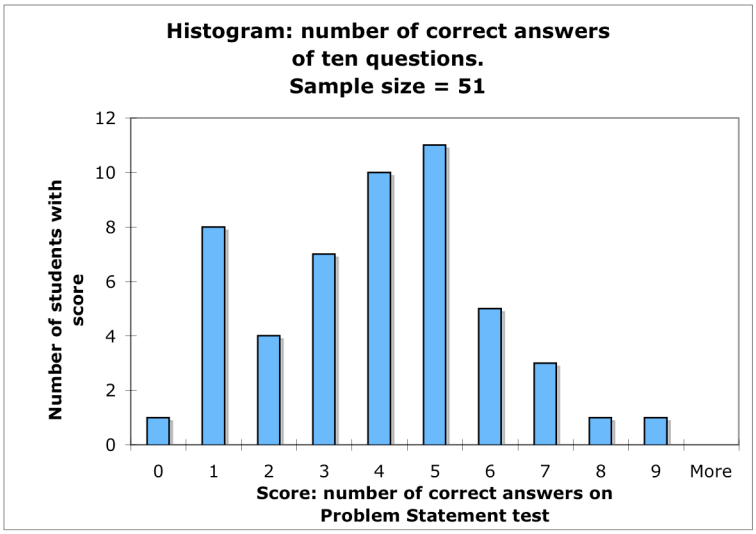
After distributing the test I read out loud, as the students presumably were following, the instructions on page 1 of the test. I then repeated in my own words that the students were *not* being asked to solve any of these problems[1] but were being asked "what does it say?"

Of 61 Geometry students there were 51 completed tests with useful results from students who were present at the times that the tests were given.

Results

I scored the tests by entering the answer letters for each student directly into an Excel spreadsheet that did the scoring automatically. In addition, the spreadsheet provided the statistics I report below.

Here is a histogram showing the frequency of each score, from 0 to 10 questions out of 10 answered correctly.



[1] Questions 5 and 6 did not quite follow the pattern I described to the students.

Here are the general statistics.. The numbers shown are the numbers of correct answers out of ten questions.

Median:	4.0
Mean:	3.9
Standard Deviation:	2.0

Exploration of Results

These results should be seen as suggestive but not definitive. There are many ways an analysis of these data can go astray; here are some that occur to me.

- The sample is too small
- The test does not measure what it purports to measure.
- The test is not representative of what the students actually encounter in school.
- The students did not take the test seriously or sabotaged their answers one way or another.
- The problem-solving model presented above is inappropriate.

Each of these factors is present to some unknown degree. Nevertheless, I find the results alarming.

The first problem statement (on page 1 of the test) is instructive. About a week before the test I spent almost half a period parsing and analyzing this problem statement for my classes. Before that presentation the students were overwhelmingly clueless about what they were being asked to provide. What I had to teach was how to parse an English sentence. Do our students learn that? I saw no evidence of such skills.

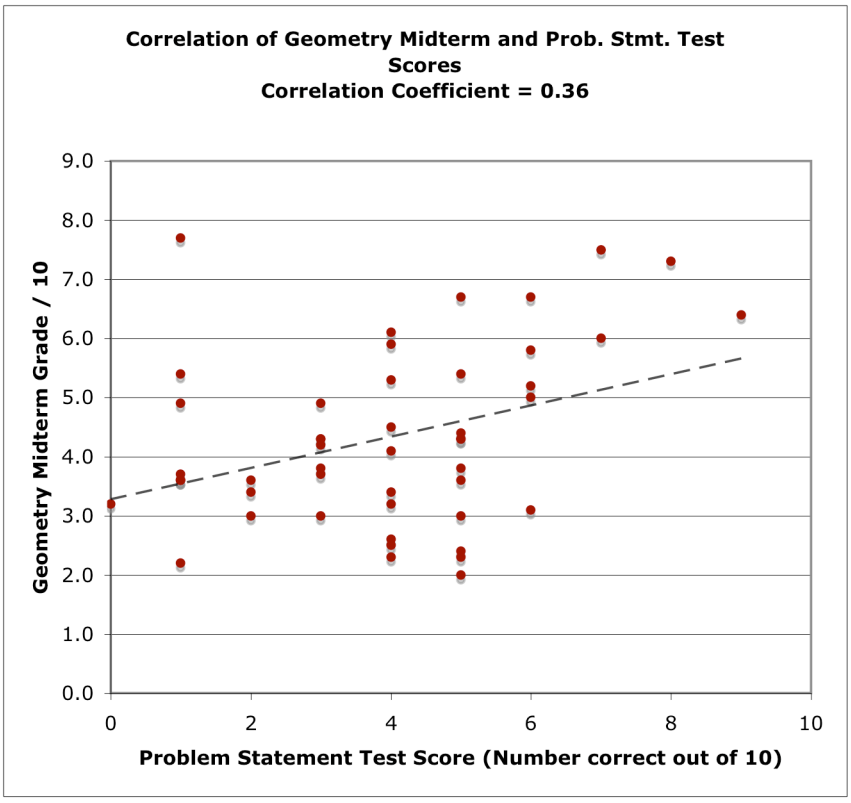
The first problem statement requires virtually no knowledge of Geometry in order to answer the three multiple-choice questions. Built into the problem statement are these facts.

- You are being asked to name a segment.
- The segment has a length.
- There is a point and a line and there is a distance between them.
- The length of the segment is that distance.

The answers to questions 1 and 2 of the test are built into the one-sentence problem statement. Roughly one third of the students answered these questions correctly. *Isn't it fair to conclude that answers to worksheet questions 15 and 16 by the other two-thirds of the students carry no useful information about their content knowledge?*

I chose questions from our Glencoe worksheets in order to raise another question: *Is the written material presented in a way that our students can understand?* For a majority of our students I believe it is not.

Because the students had just completed the midterm Geometry exams when I started to write this paper I decided to correlate their midterm results and the results of this test. Here is the scatter plot Excel provided.[2] The dotted trend line also comes from Excel. The correlation coefficient is 0.36.



The statistics that say that roughly 40% of the answers of my test were answered correctly raise this question: What fraction of the 40% knew the correct answers and what fraction just guessed correctly?

For a four-choice multiple-choice test, making one reasonable assumption along the way, you can show that, if K is the fraction of students who answered correctly because they knew the answer and S is the total fraction of students answering correctly,

$$K = \frac{4}{3}S - \frac{1}{3}$$

Substituting $S = 40\%$ into this equation yields this result.

Only 20% of the questions were answered correctly because the students knew the correct answers.

[2] Two of the 51 students had not taken the midterm exam at the time of this writing. I excluded them from the correlation analysis.

Appendix: Test with Answers

In this test you will see actual statements of problems from Geometry worksheets. These problem statements are enclosed in boxes. You are *not* being asked to solve the stated problems. Instead, you are being asked *about the meanings* of the problems statements.

Here is the form of the questions in this test.

1. A problems statement, taken from an actual worksheet, is enclosed in a box.
2. Following the box are one or more multiple-choice questions about the problem statement in that box.

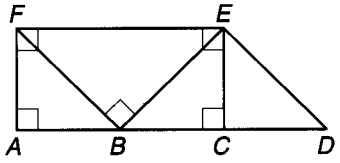
Your job is to answer the multiple-choice questions; they ask you about the *meanings* of the problem statements.

Answer the multiple-choice questions; **you do not need to solve the problems themselves.**

To answer each multiple-choice question, write the letter of the correct choice (A, B, C, or D) ***on the line to the left of the question number.***

Multiple-choice questions 1, 2 and 3 refer to this problem statement:

> ***Name the segment whose length is the distance between the point and line.***
>
> **15.** from F to $\overleftrightarrow{BE}$
>
> **16.** from D to $\overleftrightarrow{FA}$



B
31%
33%
24%
12%

_____1. What can a correct answer to each of questions 15 and 16 contain?

A. A point and a line.
B. A segment.
C. The length of a segment.
D. A line.

A
31%
10%
33%
25%

_____2. What kind of thing is the distance mentioned in the problem statement?

A. The length of a segment.
B. The measure of an angle.
C. The length of a line.
D. None of the above.

A
49%
25%
14%
12%

_____3. Identify the point and the line mentioned in the problem statement.

A. They are different for problem 15 and problem 16.
B. Your job is to find them in the figure.
C. The point is F and the line is $\overleftrightarrow{BE}$ for both problems.
D. The point is D and the line is $\overleftrightarrow{FA}$ for both problems.

Multiple-choice question 4 refers to this problem statement:

> ***Find the slopes of the lines parallel to and perpendicular to the line through the given points.***
>
> **10.** $A(2, 3)$, $B(4, 4)$
>
> **11.** $C(3, -3)$, $D(6, -5)$
>
> **12.** $E(-1, -2)$, $F(-1, 3)$
>
> **13.** $G(0, 0)$, $H(2, 7)$
>
> **14.** $J(-5, -2)$, $K(3, -2)$

D	_____4. What can a correct answer to each of questions 10-14 contain?
37%	A. A line.
14%	B. Two lines.
16%	C. A number.
33%	D. Two numbers.

Multiple-choice questions 5 and 6 refer to this statement:

> ***Each of the six faces of a cube has a different number on it. The six numbers are consecutive whole numbers. Three of these numbers are 19, 20, and 23.***

B	_____5. Of all the possible arrangements of six consecutive numbers as described in the box, what is the largest number that can appear?
41%	A. 23.
29%	B. 24.
8%	C. 25.
22%	D. None of the above.

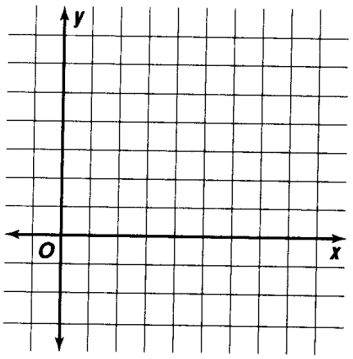
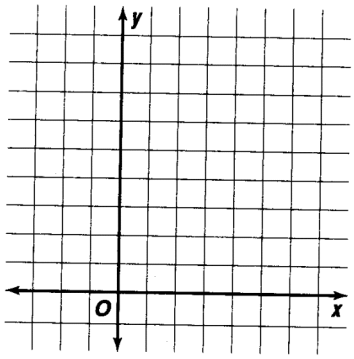
A	_____6. Of all the possible arrangements of six consecutive numbers as described in the box, what is the smallest number that can appear?
29%	A. 18.
51%	B. 19.
2%	C. 20.
18%	D. None of the above.

Multiple-choice question 7 refers to this problem statement:

> ***Two sides of a triangle are 21 and 24 inches long. Determine whether each measurement can be the length of the third side.***
>
> **11.** 3 inches **12.** 40 inches **13.** 56 inches

C	_____7. What can a correct answer to each of questions 11, 12, and 13 contain?
33%	A. A length in inches.
14%	B. Two lengths in inches.
25%	C. The word "yes" or the word "no."
27%	D. The word "yes" or the word "no," and a length in inches.

Multiple-choice question 8 refers to this problem statement:

> ***Determine the value of r so that a line through the points with the given coordinates has the given slope. Draw a sketch of each situation.***
>
> **9.** (5, 3), (r, 6); slope = 1
>
> 
>
>
> **10.** (2, r), (−2, 6); slope = $\frac{1}{2}$
>
> 
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B	_____8. What can a correct answer to each of these questions contain?
18%	A. A number.
65%	B. A number and a drawing.
12%	C. A drawing.
6%	D. Two numbers.

Multiple-choice question 9 refers to this problem statement:

Determine if a valid conclusion can be reached from the two true statements using the Law of Detachment or the Law of Syllogism. If a valid conclusion is possible, state it and the law that is used. If a valid conclusion does not follow, write no conclusion.

1. If Jim is a Texan, then he is an American.
   Jim is a Texan.

2. If Spot is a dog, then he has four legs.
   Spot has four legs.

3. If Rachel lives in Tampa, then Rachel lives in Florida.
   If Rachel lives in Florida, then Rachel lives in the United States.

4. If October 12 is a Monday, then October 13 is a Tuesday.
   October 12 is a Monday.

5. If Henry studies his algebra, then he passes the test.
   If Henry passes the test, then he will get a good grade.

C

_____9. What can a correct answer to each of these five questions contain?

6% A. Either the words "no conclusion," or a conclusion.

25% B. Either the words "no conclusion," or the name of a law.

35% C. Either the words "no conclusion," or both the name of a law and a conclusion.

33% D. A conclusion and the Law of Detachment, or a conclusion and the Law of Syllogism.

Multiple-choice question 10 refers to this problem statement:

Determine if the conjecture is true or false based on the given information. Explain your answer and give a counterexample for any false conjecture.

1. Given: noncollinear points A, B, C, and D
   Conjecture: A, B, C, and D are coplanar.

2. Given: A, B, C, and D are collinear points.
   Conjecture: $AB + BC + CD = AD$

3. Given: $\angle A$ and $\angle B$ are right angles.
   Conjecture: $\angle A \cong \angle B$

4. Given: Point C between H and V.
   Conjecture: $\angle HCV$ is a straight angle.

D

_____10. What can a correct answer to each of these four questions contain?

14% A. The word "true" or the word "false."

8% B. The word "true" or the word "false," and a conjecture.

18% C. The word "true" or the word "false," and an explanation.

61% D. The word "true" and an explanation, or the word "false" and an explanation and a counterexample.